FLOW OF A BINARY GAS MIXTURE IN A VERTICAL PLANE-PARALLEL DIFFERENTIAL-TEMPERATURE CHANNEL

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The forced flow of a binary gas mixture in a vertical plane-parallel differential-temperature channel is investigated with allowance for free convection and phase transitions of one of the components of the mixture at the walls of the channel.

Vertical differential-temperature channels [1, 2] are used in studies of the diffusiophoresis of aerosol particles; their heated wall serves as a vapor source, and the cold wall provides a surface of condensation. A gas containing aerosol particles is passed through such channels. Of utmost interest here from the point of view of diffusiophoresis work are forced-flow regimes for which the Reynolds numbers Re are smaller than the Grashof number G and its concentration analog Ar, i.e., under the conditions $\mathrm{Re} \leqslant G$ and $\mathrm{Re} \leqslant \mathrm{Ar}$. In these flow regimes, estimates of the terms in the equation of motion show that free convection must be taken into consideration. Accordingly, in the present article we give a solution for the system of hydrodynamic equations of the mixture with allowance for free convection and phase transitions of one of the components of the mixture at the walls of the channel. The system is solved for the case of a vertical channel in the form of a narrow passage confined between parallel plane surfaces.

In solving the problem we use the customary simplifying assumptions [3]; we also assume that the gas mixture in the channel is Newtonian and obeys the ideal gas laws, while the air (in a vapor—air mixture) behaves as a simple component. In this case the transport processes are described by the system of hydrodynamic equations for a binary mixture [4], which in the steady state without thermal diffusion, diffusion heat conduction, and energy dissipation through internal friction has the form

$$\nabla \rho \mathbf{v} = 0, \tag{1}$$

$$\rho \mathbf{v} \nabla c_i = \nabla \rho D \nabla c_i, \tag{2}$$

$$\rho c_p \mathbf{v} \nabla T = \nabla k \nabla T, \tag{3}$$

$$\rho (\mathbf{v} \nabla) \mathbf{v} = -\nabla P + \nabla \mu \nabla \mathbf{v} + \rho_0 g (\beta_1 \delta T + \beta_2 \delta c_1) \gamma. \tag{4}$$

We use a Cartesian coordinate system with the origin situated on one of the vertical plane surfaces of the channel (either the hot or the cold wall), the x axis directed vertically upward (parallel to the channel axis), and the y axis directed perpendicular to those surfaces.

In accordance with the investigated problem, a constant temperature and a constant concentration of the lighter component, which is evaporated by the heated (hot) surface, are maintained on that surface. On the opposite cooled (cold) vertical plane surface of the channel as well, a constant but lower temperature is maintained. In this case the lighter component of the mixture is condensed (absorbed), and so it maintains a constant value of the concentration, which corresponds to the saturation vapor pressure at the temperature of the cooled surface. In the adopted coordinate system, therefore, the boundary conditions of the problem are

$$\text{at } y = 0 \quad T(x,\ 0,\ z) = T(0),\ c_i(x,\ 0,\ z) = c_i(0),\ V_x(x,\ 0,\ z) = 0, \tag{5}$$

$$\text{at } y = h \quad T(x,\ h,\ z) = T(h),\ c_i(x,\ h,\ z) = c_i(h),\ V_x(x,\ h,\ z) = 0. \tag{6}$$

By the symmetry of the channel and the boundary conditions, for steady longitudinal (along the vertical axis of the channel) laminar forced flow (at a sufficient distance from

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the inlet to the differential-temperature channel) all quantities (except the pressure P) will depend on the one coordinate y. In this case the system of equations (1)-(4) acquires the form

$$\frac{d}{dy}\rho V_y=0, \tag{7}$$

$$\rho V_y\frac{dc_i}{dy}=\frac{d}{dy}\rho D\frac{dc_i}{dy}, \tag{8}$$

$$\rho V_y c_p\frac{dT}{dy}=\frac{d}{dy}k\frac{dT}{dy}, \tag{9}$$

$$\rho V_y\frac{dV_x}{dy}=-\frac{dP}{dx}+\frac{d}{dy}\mu\frac{dV_x}{dy}+\rho_0 g(\beta_1\delta T+\beta_2\delta c_1). \tag{10}$$

The equations of motion in the directions of the y and z axes (which are perpendicular to the flow), as in [3], are disregarded, and it is assumed that $\partial P/\partial y = 0$ and $\partial P/\partial z = 0$.

To further simplify the system of equations (7)-(10) we integrate the continuity and diffusion equations (7) and (8). After one integration of (7) we obtain

$$\rho V_y = C_1, \tag{11}$$

where C_1 is a constant of integration. Integrating (8) with regard for (11), we have

$$\rho V_y c_1=\rho D\frac{dc_1}{dy}+C_2, \tag{12}$$

$$\rho V_y c_2=\rho D\frac{dc_2}{dy}+C_3, \tag{13}$$

where C_2 and C_3 are constants of integration. Inasmuch as the second component of the mixture (air) is inert, $C_3 = 0$. From (13) we then find the perpendicular component of the mass-velocity of the mixture:

$$V_y=\frac{D}{c_2}\frac{dc_2}{dy}. \tag{14}$$

Expression (14) coincides with the expression for the Stefan flow velocity, and so the given component V_y of the mass velocity of the binary gas mixture is the Stefan flow velocity.

Bearing in mind that $c_1 + c_2 = 1$ and $C_3 = 0$, from Eqs. (11)-(13) we obtain $C_1 = C_2 = \rho V_y$, so that

$$V_y = C_1/\rho. \tag{15}$$

Making use of expression (11) and the fact that $C_1 = C_2$, we rewrite Eq. (12) in the form

$$\frac{dc_1}{dy}-\frac{C_1}{\rho D}c_1+\frac{C_1}{\rho D}=0. \tag{16}$$

Equation (16) describes the concentration distribution of the lighter component (vapor) in the channel in the y direction (i.e., over the width of the channel). From (16) we find

$$C_1=\frac{\rho D}{(c_1-1)}\frac{dc_1}{dy}. \tag{17}$$

Taking the values of D and ρ in (17) as numerically equal to the average values $\bar{D}$ and $\bar{\rho}$ over the width of the channel, after integration we obtain the value of the constant

$$C_1=\frac{\bar{\rho}\,\bar{D}}{h}\ln\frac{c_1(h)-1}{c_1(0)-1}. \tag{18}$$

We introduce the notation

$$\Phi=\frac{C_1}{\bar{\rho}\,\bar{D}}=\frac{\ln\frac{c_1(h)-1}{c_1(0)-1}}{h}. \tag{19}$$

Using the value obtained for the constant C_1 in expression (15), we find a new expression for the Stefan flow velocity or the transverse component of the mass velocity of the gas mixture:

$$V_y = \frac{\bar{\rho}\,\bar{D}}{\rho h} \ln \frac{c_1(h) - 1}{c_1(0) - 1}. \tag{20}$$

From (20), averaging over the width of the channel, we obtain an expression for the average transverse component of the velocity of the gas mixture or the average value of the Stefan flow velocity:

$$\bar{V}_y = \frac{\bar{D}}{h} \ln \frac{c_1(h) - 1}{c_1(0) - 1}. \tag{21}$$

Returning to the system of equations (7)-(10), we transform the other two equations (9) and (10). Using the solutions of the continuity equation (11), we transform the convective heat-conduction equation (9) to the form

$$C_1 c_p \frac{dT}{dy} = \frac{d}{dy} k \frac{dT}{dy}.$$

Setting c_p equal to the average value of the specific heat of the mixture over the width of the channel, $c_p = \bar{c}_p$, after one integration we obtain a first-order differential equation describing the temperature distribution in the y direction (over the width of the channel):

$$\frac{dT}{dy} - \frac{C_1 \bar{c}_p}{k} T + \frac{C_5}{k} = 0, \tag{22}$$

where C_5 is a constant of integration.

Bearing in mind the solution of the continuity equation (11) and grouping terms in Eq. (10), we obtain

$$\frac{d}{dy} \mu \frac{dV_x}{dy} - C_1 \frac{dV_x}{dy} = \frac{dP}{dx} - \rho_0 g (\beta_1 \delta T + \beta_2 \delta c_1). \tag{23}$$

The resulting equation (23), in contrast with (10), is a linear differential equation.

We perform a single integration of Eq. (23) with respect to y:

$$\frac{dV_x}{dy} - \frac{C_1}{\mu} V_x = \frac{dP}{dx} \frac{y}{\mu} - \frac{\rho_0 g}{\mu} \int (\beta_1 \delta T + \beta_2 \delta c_1)\, dy + \frac{C_4}{\mu}.$$

Thus, after suitable transformations and integration we obtain the simplified system of equations

$$\frac{dc_1}{dy} - \frac{C_1}{\rho D} c_1 + \frac{C_1}{\rho D} = 0, \tag{24}$$

$$\frac{dT}{dy} - \frac{C_1 \bar{c}_p}{k} T + \frac{C_5}{k} = 0, \tag{25}$$

$$\frac{dV_x}{dy} - \frac{C_1}{\mu} V_x = \frac{dP}{dx} \frac{y}{\mu} - \frac{\rho_0 g}{\mu} \int (\beta_1 \delta T + \beta_2 \delta c_1)\, dy + \frac{C_4}{\mu}. \tag{26}$$

By numerical integration of the system of equations (24)-(26) it is possible to find the distributions of the concentration of the lighter component (vapor), the temperature, and the vertical component of the particle velocity of the gas mixture in the channel for variable transfer coefficients and variable density.

To obtain an analytical solution of the system of equations (24)-(26) subject to the boundary conditions (5) and (6) we first consider the processes of diffusion and heat conduction within the framework of their approximate description.

Analyzing the diffusion processes, we set the values of ρ and D in Eq. (24) numerically equal to the average values $\bar{\rho}$ and $\bar{D}$ over the width of the channel. Then the solution of Eq. (24) satisfying the given boundary conditions (5) and (6) and describing the concentration distribution of the lighter component (vapor) in the channel in the y direction is

$$c_1(y) = \frac{[c_1(0) - c_1(h)][\exp(\Phi y) - \exp(\Phi h)]}{1 - \exp(\Phi h)} + c_1(h). \tag{27}$$

Determining the expression for the average concentration $\bar{c}_1$ of the lighter component over the width of the channel (by averaging (27) over y) and introducing the notation

$a = 1/[1 - \exp(\Phi h)]$, $b = -1/\Phi h$, $\Delta c_1 = c_1(0) - c_1(h)$, we obtain an analytical expression for the deviation of the concentration of the average value $\bar{c}_1$:

$$\delta c_1(y) = \Delta c_1 [a \exp(\Phi y) - b]. \tag{28}$$

Next we consider heat-conduction processes. In Eq. (25) we set the quantity k equal to the average value $\bar{k}$ over the width of the channel. Then the solution of Eq. (25) satisfying the given boundary conditions (5) and (6) and describing the temperature distribution in the channel in the y direction is

$$T(y) = \frac{[T(0) - T(h)][\exp(\Phi_1 y) - \exp(\Phi_1 h)]}{1 - \exp(\Phi_1 h)} + T(h), \tag{29}$$

where $\Phi_1 = C_1 \bar{c}_p / \bar{k}$.

Determining the expression for the average temperature over the width of the channel by averaging the solution (29) over y, and introducing the notation $A = 1/[1 - \exp(\Phi_1 h)]$, $B = -1/\Phi_1 h$, $\Delta T = T(0) - T(h)$, we find the deviation of the temperature from the average value $\bar{T}$:

$$\delta T(y) = \Delta T [A \exp(\Phi_1 y) - B]. \tag{30}$$

We consider the expression (26) for the longitudinal vertical component V_x of the particle velocity and substitute into its right-hand side the analytical expressions (28) and (30) obtained for $\delta c_1(y)$ and $\delta T(y)$. Integrating on the right-hand side of Eq. (26), we obtain

$$\frac{dV_x}{dy} - \frac{C_1}{\mu} V_x = \frac{dP}{dx}\frac{y}{\mu} - \frac{\rho_0 g \beta_1 \Delta T}{\mu}\left[\frac{A \exp(\Phi_1 y)}{\Phi_1} - By\right] - \frac{\rho_0 g \beta_2 \Delta c_1}{\mu}\left[\frac{a \exp(\Phi y)}{\Phi} - by\right] + \frac{C_4}{\mu}. \tag{31}$$

The general solution of Eq. (31) has the form

$$V_x = \left(\int\left\{\frac{dP}{dx}\frac{y}{\mu} - \frac{\rho_0 g \beta_1 \Delta T}{\mu}\left[\frac{A \exp(\Phi_1 y)}{\Phi_1} - By\right] - \frac{\rho_0 g \beta_2 \Delta c_1}{\mu}\left[\frac{a \exp(\Phi y)}{\Phi} - by\right] + \frac{C_4}{\mu}\right\}\right.$$
$$\left. \times \exp\left(-C_1 \int \frac{dy}{\mu}\right) dy + C_6\right) \exp\left(C_1 \int \frac{dy}{\mu}\right), \tag{32}$$

where C_6 is a constant of integration.

To simplify the solution (32) we set the dynamic viscosity equal to the average value over the width of the channel, $\mu = \bar{\mu}$, and introduce the notation

$$Q_1 = (g\beta_1\rho_0)/\bar{\mu}, \quad Q_2 = (g\beta_2\rho_0)/\bar{\mu}.$$

Then the solution satisfying the boundary conditions (5) and (6) of the problem takes the form

$$V_x = -\frac{\frac{dP}{dx} + \bar{\mu} Q_1 B \Delta T + \bar{\mu} Q_2 b \Delta c_1}{\bar{\mu}\left(\frac{C_1}{\bar{\mu}}\right)}\left\{(y - h) + \frac{h\left[\exp\left(\frac{C_1}{\bar{\mu}} h\right) - \exp\left(\frac{C_1}{\bar{\mu}} y\right)\right]}{\left[\exp\left(\frac{C_1}{\bar{\mu}} h\right) - 1\right]}\right\} - \frac{Q_1 A \Delta T}{\Phi_1\left(\Phi_1 - \frac{C_1}{\bar{\mu}}\right)} \times$$

$$\times \left\{[\exp(\Phi_1 y) - \exp(\Phi_1 h)] + \frac{[\exp(\Phi_1 h) - 1]\left[\exp\left(\frac{C_1}{\bar{\mu}} h\right) - \exp\left(\frac{C_1}{\bar{\mu}} y\right)\right]}{\left[\exp\left(\frac{C_1}{\bar{\mu}} h\right) - 1\right]}\right\} -$$

$$- \frac{Q_2 a \Delta c_1}{\Phi\left(\Phi - \frac{C_1}{\bar{\mu}}\right)}\left\{[\exp(\Phi y) - \exp(\Phi h)] + \frac{[\exp(\Phi h) - 1]\left[\exp\left(\frac{C_1}{\bar{\mu}} h\right) - \exp\left(\frac{C_1}{\bar{\mu}} y\right)\right]}{\left[\exp\left(\frac{C_1}{\bar{\mu}} h\right) - 1\right]}\right\}. \tag{33}$$

In analyzing and solving the equation of motion (10) it has been assumed that the equations of motion in the directions of the y and z axes have the form $\partial P/\partial y = 0$, $\partial P/\partial z = 0$. In this case the pressure gradient in the x direction has a constant value, i.e., $\partial P/\partial x = \text{const}$. The resulting solution (33) can be used to refine the value of the gradient and to determine the pressure distribution P in the channel analytically. However, the cumbersomeness of the derived expressions and the simplicity of their derivation prompt their omission from the article.

Thus, as a result of the investigation we have obtained analytical expressions (20) and (33) for both components of the mass velocity V_x and V_y, along with expressions (27) and (29) describing the distributions of the concentration c_1 and the temperature T in the channel.

To illustrate the influence of free convection and Stefan flow on the profile of the longitudinal vertical component of the mass velocity Fig. 1 gives the results of calculations according to the derived expressions for a fixed wall temperature regime (temperature of the cold wall 27°C, temperature of the hot wall 60°C) and various flow regimes of the vapor—air mixture. Curve 1 gives the profile of the dimensionless longitudinal vertical component of the mass velocity $V_x/\overline{V}_x$ (where $\overline{V}_x$ is the average value of V_x in the channel) for the forced upward vertical flow of a vapor—air mixture when the Grashof number is equal to the Reynolds number, i.e., under the condition $Re/G = 1$. Also shown in the figure for comparison is the Poiseuille flow profile (curve 2), again in dimensionless form. A comparison of curves 1 and 2 shows that the profile of the vertical component of the mass velocity in the given situation is close to the Poiseuille form. The observed shift of the profile to the right is attributable to the influence of Stefan flow and is directed toward the cold wall of the channel at $y = h$. Curve 3 in Fig. 1 represents the profile of the vertical component of the mass velocity (in dimensionless form) for $Re/G = 0.1$. It is evident from a comparison of curves 3 and 2 that under these conditions there is already an appreciable departure of the vertical component of the mass velocity from the Poiseuille profile under the influence of free convection. Here the profile of the vertical component of the mass velocity has now shifted to the left toward the hot wall of the channel, and the influence of Stefan flow is completely suppressed by the influence of free convection.

Finally, curve 4 in Fig. 1 represents the profile of the longitudinal vertical component of the mass velocity (also in dimensionless form) under conditions such that $Re/G = 0.01$. It is evident from a comparison of curves 4 and 2 that under these conditions the profile of the longitudinal vertical component differs from the Poiseuille form and near one of the walls of the channel there is a reverse-flow zone (i.e., motion opposite to the forced flow direction). Here the observed distortions of the profile of the longitudinal vertical component of the mass velocity are associated entirely with the influence of free convection.

Thus, in the upward flow of a binary vapor—air mixture the distorting effects of Stefan flow and free convection on the profile of the vertical component of the mass velocity are counteractive. When $Re/G \sim 1$, the influence of free convection is absent, and the observed distortion (shift) of the profile is entirely attributable to the influence of Stefan flow. Under conditions such that $Re/G < 1$ the influence of free convection sets in, which initially, as the value of the ratio Re/G is decreased, produces a gradual compensation of the Stefan flow effect, ultimately suppressing it altogether, and is then accompanied by a change in the entire flow profile and the onset of a reverse-flow zone.

In conclusion we consider the inertialess motion of aerosol particles under the investigated conditions. Figure 2 shows the trajectories of the aerosol particles in dimensionless form (in the dimensionless variables x/l_i and y/h, where l_i is the limiting precipitation length in the i-th test and h is the distance between the vertical differential-temperature surfaces) when the motion of the particles is initiated directly from the hot surface of the channel; the curves are calculated on the basis of Eqs. (20) and (33) by integrating the equations of motion. Curve 1 represents the aerosol particle trajectory in the investigated channel for flow regimes and wall temperature regimes corresponding to conditions such that $Re/G \sim 1$, curve 2 represents the particle trajectory for $Re/G \sim 0.1$, and curve 3 for $Re/G \sim 0.01$.

It is evident from the curves that for flow regimes and wall temperature regimes corresponding to the conditions $Re/G \sim 0.01$ considerable distortion is observed in the trajectory of the aerosol particles as a result of free convection. This fact must be taken into account in measurements of the diffusiophoresis and thermophoresis rates from the critical mass flow of gas through a duct or from the characteristic time constant τ. The correct values of the diffusiophoresis rate in this case correspond, not to zero passage of aerosol particles through the channel, but to ~36% passage.

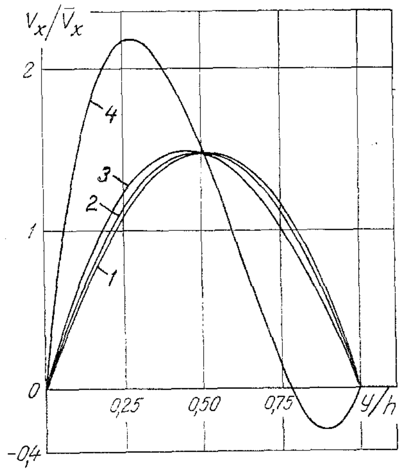

Fig. 1

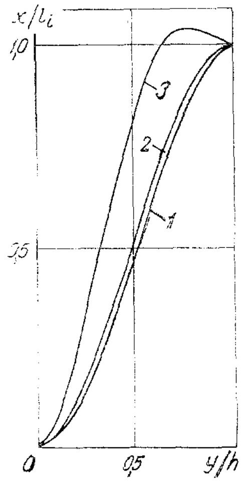

Fig. 2

Fig. 1. Profile of the longitudinal vertical component of the mass velocity in dimensionless form for a cold-wall temperature of 27°C and a hot-wall temperature of 60°C. 1) Re/G = 1; 2) Poiseuille flow profile; 3) Re/G = 0.1; 4) Re/G = 0.01.

Fig. 2. Trajectories of aerosol particles. 1) Re/G = 1; 2) Re/G = 0.1; 3) Re/G = 0.01.

NOTATION

$c_i = \rho_i/\rho$, concentration of i-th component of gas mixture, i = 1 for the lighter component, and i = 2 for the heavier component of the mixture; $\rho = \rho_1 + \rho_2$, density of binary gas mixture; ρ_1, ρ_2, densities of first and second components of mixture; ρ_0, density of mixture at average values of the temperature and concentration; T, temperature; **v**, average mass-velocity vector of mixture at a given point of space; V_x, component of average mass-velocity vector along x axis; $\bar{V}_x$, average value of V_x in channel; V_y, component of average mass-velocity vector along y axis; $\bar{V}_y$, average value of V_y in channel; D, interdiffusion coefficient of mixture components; $\bar{D}$, average value of D; k, thermal conductivity of mixture; $\bar{k}$, average value of k; μ, dynamic viscosity of mixture; $\bar{\mu}$, average value of μ; c_p, specific heat of mixture at constant pressure; $\bar{c}_p$, average value of c_p; P, pressure relative to hydrostatic value corresponding to density ρ_0; g, acceleration of gravity; γ, unit vector in upward vertical direction; δT, deviation from average temperature; δc_1, deviation from average concentration; β_1, thermal expansion coefficient of mixture; $\beta_2 = 1/\rho_0(\partial\rho/\partial c_1)_{T,p}$ expresses the density of the mixture as a function of the concentration c_1 of the lighter component; C_1, C_2, C_3, C_4, C_5, C_6, constants of integration; Φ, function defined in (19); $\Phi_1 = C_1\bar{c}_p/\bar{k}$, $A = 1/[1 - \exp(\Phi_1 h)]$, $B = -1/(\Phi_1 h)$, $\Delta T = T(0) - T(h)$; h, width of channel (distance between plane vertical differential-temperature surfaces); $a = 1/[1 - \exp(\Phi h)]$, $b = -1/(\Phi h)$, $\Delta c_1 = c_1(0) - c_1(h)$, $Q_1 = g\beta_1\rho_0/\bar{\mu}$, $Q_2 = g\beta_2\rho_0/\bar{\mu}$, $Re = \bar{V}_x h\rho_0/\bar{\mu}$, $G = g\beta_1 h^3\Delta T\rho_0^2/\bar{\mu}^2$, $Ar = g\beta_2 h^3\Delta c_1\rho_0^2/\bar{\mu}^2$.

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